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Method for analyzing the stability of information transfer between unmanned aerial vehicles in the formation

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ABSTRACT

The use of a formation consisting of adaptive autonomous mobile agents allows solving a wide range of tasks that are often beyond the capabilities of individual agents. A multi-agent formation is a complex high-order dynamic system, so analyzing the stability of such a system is a complex task. At present, the problem of estimating the stability of a formation formed by substantially nonlinear high-order agents with variable dynamic parameters is not sufficiently considered. This task is particularly important for a formation that is affected by complex unstable environmental conditions, in particular for the formation of unmanned aerial vehicles (UAVs). A method for analyzing the stability of formations of nonlinear agents with different types and orders of transfer function has been developed for studying information exchange in UAV networks. The new approach is based on the use of the Popov frequency criterion and the piecewise linear approximation of the hodograph. A computational experiment was performed to analyze the stability of a formation with transfer functions of various types and orders from the 1st to the 10th. The conducted studies revealed a significant difference in the calculated boundary coefficients of formation stability in the linear and nonlinear modes, which confirms the need to analyze the nonlinear stability under the influence of strong destabilizing influences on the formation.

KEYWORDS: multi-agent formation; mobile agent formation; unmanned aerial vehicle; UAV; data transmission; stability; Nyquist criterion; Popov criterion.

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Método para analizar la estabilidad de la transferencia de información entre vehículos aéreos no tripulados en la formación

RESUMEN

El uso de una formación que consiste en agentes móviles autónomos adaptativos, permite resolver una amplia gama de tareas que a menudo están más allá de las capacidades de los agentes individuales. Una formación de múltiples agentes es un sistema dinámico complejo de alto orden, por lo que analizar la estabilidad de dicho sistema es una tarea compleja. En la actualidad, el problema de estimar la estabilidad de una formación integrada por agentes de alto orden sustancialmente no lineales con parámetros dinámicos variables, no se considera suficientemente. Esta tarea es particularmente importante para una formación que se ve afectada por condiciones ambientales complejas e inestables, en particular para la formación de vehículos aéreos no tripulados (UAV). Se ha desarrollado un método para analizar la estabilidad de las formaciones de agentes no lineales con diferentes tipos y órdenes de función de transferencia para estudiar el intercambio de información en redes UAV. El nuevo enfoque se basa en el uso del criterio de frecuencia de Popov y la aproximación lineal por partes del hodógrafo. Se realizó un experimento computacional para analizar la estabilidad de una formación con funciones de transferencia de varios tipos y órdenes del 1 al 10. Los estudios realizados revelaron una diferencia significativa en los coeficientes límite calculados de la estabilidad de la formación en los modos lineal y no lineal, lo que confirma la necesidad de analizar la estabilidad no lineal bajo la influencia de fuertes influencias desestabilizadoras en la formación.

PALABRAS CLAVE: formación de múltiples agentes; formación de agentes móviles; vehículos aéreos no tripulados; UAV; transmisión de datos; estabilidad; criterio de Nyquist; criterio de Popov.

Introduction

Using a formation consisting of adaptive autonomous mobile agents allows to solve a wide range of tasks that are often beyond the capabilities of individual agents. In particular, such tasks include positioning mobile robots in production, automating traffic, monitoring the environment with unmanned aerial vehicles (UAVs), and others. Cooperation of individual agents in the formation is based on the interaction protocols of network systems (Amelina, 2011; Wu, 2007).

A multi-agent formation is a complex dynamic system of high order, so analyzing the stability of such a system is a complex task. Existing approaches to assessing the stability of

mobile agent formations are based on a number of simplifying assumptions. Thus, in (Fax & Murray, 2004) the assumption is made about the linearity and identity of the dynamics models of the agents in a formation. The stability analysis of the formation in this paper is based on checking the stability condition of an individual agent using the Nyquist frequency criterion, taking into account the spectrum of the Laplace matrix describing the graph of information transfer between agents. Synchronization in networks of passive agents (including nonlinear ones) has been studied in a number of papers, for example (Proskurnikov, 2014). Most of the known results assume that the graph of agent formation is constant, or that there is a positive delay (dwell-time) between changes. In (Proskurnikov, 2014), the problem of consensus of nonlinear agents of the 2nd order (double integrators) is investigated, in addition, nonstationary communication functions are considered. A number of papers consider the problems of adaptive synchronization of interconnected dynamic subsystems (Chopra & Spong, 2006; Arcak, 2007) and adaptive management of agents with significantly different dynamic properties. The influence of the bandwidth of the information exchange channel between agents on the stability of the agent system and its dynamic properties as a whole is studied (Andrievsky et al., 2010; Amelin, et al., 2019). At the same time, the problem of estimating the stability of a formation formed by essentially nonlinear high-order agents with variable dynamic parameters is not sufficiently considered. This problem is particularly relevant for a formation that is affected by complex unstable environmental conditions, in particular, for a UAV formation.

The aim of this work is to develop a method for analyzing the stability of high-order nonlinear agent formations with variable dynamic parameters for information exchange in UAV networks.

1. Method for analyzing the absolute stability of the UAV formation as high-order nonlinear agents

Consider the UAV formation as a network of interconnected dynamic subsystems represented in the Lurie form (a dynamic linear unit (LU) with a static non-linear unit (NLU) in feedback). According to (Fax & Murray, 2004), K(s) stabilizes the dynamics of agent formation with many inputs and many outputs of P(s) if

$$\rho(W(j\omega)) < M^{-1} \forall \omega \in (-\infty; \infty),$$

where $W(s) = P(s)K(s)(I + P(s)K(s))^{-1}$, *I* is the unit matrix. This condition allows us to check the stability of the formation of linear agents according to the Nyquist criterion. When exposed to destabilizing perturbations on formation agents operate in a significantly nonlinear regime, so the condition of Nyquist stability criterion can be broken, and it is necessary to study the stability of nonlinear system by Popov criterion. According to the Popov criterion (Krasovsky, 1987; Popov, 1970), for absolute stability of the equilibrium of a nonlinear system with a stable LU, the existence of a real *g* for which the condition is satisfied is sufficient

$$\forall \omega \ge 0 : \operatorname{Re}[(1 + j\omega g)W(j\omega)] > -1/k, \qquad (1)$$

where *k* is the angle of absolute stability, $W(j\omega) = \frac{A_1(\omega) + jA_2(\omega)}{B_1(\omega) + jB_2(\omega)}$ is the complex transfer

function of the LU, and *I* is order of the LU. The highest and lowest k values, at which the condition (1) is met, determine, respectively, the lower and upper bounds of the area of stable operation of the agent in a formation.

Let's divide the complex transfer function of a LU into real and imaginary parts:

$$W(j\omega) = W_R(\omega) + jW_I(\omega), \qquad (2)$$

where $W_R(\omega) = \frac{A_1(\omega)B_1(\omega) + A_2(\omega)B_2(\omega)}{B_1^2(\omega) + B_2^2(\omega)}$ is real frequency characteristic of the LU, $W_I(\omega) = \frac{A_2(\omega)B_1(\omega) - A_1(\omega)B_2(\omega)}{B_1^2(\omega) + B_2^2(\omega)}$ is imaginary frequency characteristic of the LU, polynomials $A_{1,2}(\omega) = \sum_{i=0}^{I} A_{1,2i}(\omega)$ and $B_{1,2}(\omega) = \sum_{i=0}^{I} B_{1,2i}(\omega)$ are defined according to the expressions:

$$A_{1i}(\omega) = \operatorname{Re}[A_i(j\omega)] = \alpha_{4i}\omega^{4i} - \beta_{4i+2}\omega^{4i+2},$$

$$B_{1i}(\omega) = \operatorname{Re}[B_i(j\omega)] = \beta_{4i}\omega^{4i} - \beta_{4i+2}\omega^{4i+2},$$

$$A_{2i}(\omega) = \operatorname{Im}[A_i(j\omega)] = \alpha_{4i+1}\omega^{4i+1} - \alpha_{4i+3}\omega^{4i+3},$$

$$B_{2i}(\omega) = \operatorname{Im}[B_i(j\omega)] = \beta_{4i+1}\omega^{4i+1} - \beta_{4i+3}\omega^{4i+3},$$

where α_i , β_i are coefficients of the polynomials in the numerator and denominator of the LU transfer function. Dividing the LU transmission coefficient into real and imaginary parts allows a simple geometric interpretation of the Popov criterion.

We introduce a modified complex transfer function

$$W^{*}(j\omega) = W_{R}(\omega) + jW_{I}^{*}(\omega), \qquad (3)$$

where $W_I^*(\omega) = \omega W_I(\omega)$.

By converting (1) considering (3), we obtain a sufficient condition of absolute stability in the form $W_R(\omega) - gW_I^*(\omega) > -1/k$. For the boundary values of the coefficient k, the condition takes the form of equation (the Popov line equation)

$$W_{R}(\omega) - gW_{I}^{*}(\omega) = -1/k.$$
(4)

The line defined by equation (4) passes through the point -1/k on the real axis with a slope of 1/g.

To conduct an analytical study of the absolute stability of the formation, it is necessary to obtain an expression of the Popov line for a specific type of LU $W(j\omega)$. It is not possible to solve the problem in a generalized form for an arbitrary type and order of LU. This difficulty arises from the non-linear nature of the left part of equation (4) and the presence of two unknowns g and k. Approximation of the frequency characteristics of the LU based on continuous piecewise linear functions (CPLF) (Kurilov & Romanov, 2002) allows us to linearize (4), eliminate the unknown g, and conduct an analytical study of the absolute stability of the formation in general.

Let's set the following approximation parameters: variable range from ω_0 to ω_N , N is maximum number of approximation nodes, *n* and *m* are current numbers of approximation nodes. Frequency characteristics change most quickly in the region of small values and slowly in the region of large values. Thus, in order to reduce the error, the nodes are arranged in an exponential manner. Expressions of lines that approximate the left part (4) in the current nodes will take the form

$$W_{I_{m,n}}^{*}(W_{R}) = g_{m,n}(W_{R} - b_{m,n}), \qquad (5)$$

where $g_{m,n} = (W_{I_m}^* - W_{I_n}^*)/(W_{R_m} - W_{R_n})$ are angular coefficients, $W_{I_{m,n}}^* = W_{I}^*(\omega_{m,n})$, $W_{R_{m,n}} = W_R(\omega_{m,n})$, abscissas of the approximating straight lines are defined as

$$b_{m,n} = W_{R_m} - W_{I_{m,n}}^* / g_{m,n}.$$
(6)

As a result, we get N^2 coefficients $b_{m,n}$. Among the obtained abscissa values, it is necessary to exclude those that are located outside the interval $\omega_n \div \omega_m$ and are "false". To do

this, include the CPLF $Q_{m,n}(\vartheta) = K_{\sigma} \sum_{\lambda=0}^{1} \sum_{\gamma=0}^{1} (-1)^{\lambda+\gamma} \left| \vartheta + \vartheta_n - \vartheta_m (1-\gamma) - \frac{\lambda}{2K_{\sigma}} \right|$, where K_{σ} is the steepness of the side components of the inclusion function. Function $Q_{m,n}(\vartheta)$ takes value 1 if its argument belongs to the $[\omega_n; \omega_m]$ section, and 0 otherwise.

The corresponding inclusion function for "false" abscissa values is zero, and for true values $Q_{m,n}(b_{m,n}) = 1$. To exclude "false" values of $b_{m,n}$, it is enough to multiply (6) by $Q_{m,n}(b_{m,n})$

$$b_{m,n}^* = b_{m,n} Q_{m,n}(b_{m,n}).$$
⁽⁷⁾

The boundary values of k for each true abscissa are obtained by substituting (7) in the right part (4)

$$\mathbf{k}_{m,n} = -1/b_{m,n}^{*}.$$
 (8)

Let's denote N_2^{low} and N_2^{up} - the lower and upper bounds of the range of N² values in which the formation of agents remains stable. That is, the area of stability is a segment $N_2^{low} \le N_2 \le N_2^{up}$.

To find the boundaries of the area of absolute stability, it is necessary to select one negative and one positive nearest to zero from all values (8).

The lower bound of N₂ is defined as the maximum of all negative values $k_{m,n}$

$$\widetilde{N}_{2}^{low} = \max\left\{k_{m,n}\left[1 - \widetilde{q}\left(k_{m,n}\right)\right]\right\},\tag{9}$$

where $\tilde{q}(\vartheta) = \frac{1}{2\Delta} \left[|\vartheta + \Delta| - |\vartheta| + \Delta \right]$ is the inclusion CPLF, which takes value 1 for $\vartheta \ge 0$ and 0 for $\vartheta < 0$. The multiplier $1 - \tilde{q}(\mathbf{k}_{m,n})$ in (9) excludes positive roots.

The upper bound of N₂ corresponds to the minimum of all positive values $k_{m,n}$

$$\widetilde{N}_{2}^{up} = \min\left\{N_{2k}\widetilde{q}(k_{m,n})\right\}.$$
(10)

As an example, we calculate the area of absolute stability of the agent formation, when the LU is described by the transfer function of the low-pass filter (LPF) of the 5th order $W(p) = 1/(1 + Tp)^5$, where *T* is the LU time constant.

The real and imaginary frequency characteristics of LU are obtained by expression (2) based on polynomials $A_{1,2}(\omega)$ and $B_{1,2}(\omega)$. Take the time constant of LU equal to T=1 s and make an approximation of the frequency characteristics in the range of variables

 $\omega_0 = 0.01 \text{ s}^{-1}$, $\omega_N = 7 \text{ s}^{-1}$, N=30. From all N²=900 abscissas (6), we determine the true ones (7). According to (9) and (10), $\tilde{N}_2^{low} = -1.006$ and $\tilde{N}_2^{up} = 2.9$.

Figure 2 shows the usual W and modified W^* hodographs of the frequency characteristic of the LU under consideration, approximated by the CPLF. Abscissa of approximating lines corresponding to the boundaries of the absolute stability region,

 $\tilde{b}_{low} = -1/\tilde{N}_2^{low} = 0.994$, $\tilde{b}_{up} = -1/\tilde{N}_2^{up} = -0.345$. The Popov lines for $\tilde{N}_2^{low,up}$, indicated in Fig. 2 as $W_{I_{low,up}}^*$, are obtained by (5).

The stability of a formation formed by linear agents, in accordance with (Yakovlev, 2003; Voronov, 1986), is determined by the condition that the Nyquist hodograph intersects the abscissa axis $(W_I(\omega)=0)$, the boundary values are equal to $N_2^{low} = -1$, $N_2^{up} = 2.885$, that is, the stability region of the formation of linear agents of the 5th order represents a segment $-1 \le N_2 \le 2.885$



Fig. 1. Regular (W) and modified (W^*) hodographs of the 5th order agent formation

From the comparison of hodographs in Fig. 2 it is seen that the regions of stability of the formation of linear agents and absolute stability of the formation of nonlinear agents coincide. A small error of $\tilde{N}_2^{low,up}$ (less than 1%) is caused by an error in the approximation of frequency characteristics. Similarly, the stability of formations of other orders is analyzed.

In the case of the 5th-order LPF considered above, the stability regions of the forms in the linear and nonlinear modes of the agents' functioning coincide. It is also possible that they differ for some types of LU. As an example, we can also calculate the area of absolute stability of the formation, the transfer function of which is described by a complex filter of the 4th order. The filter consists of sequentially connected integrating, inertial-integrating, and oscillating links. Transfer function of the LU formation

$$W(p) = \frac{1}{p(1+Tp)(\beta + 2\xi Tp + T^2 p^2)},$$
(11)

where *T* is time constant of LU. We assume $T=\xi=1$, $\beta=10$.

After calculating the LU coefficients α_i , β_i , we get the real and imaginary frequency response of the LU by substituting these coefficients in (2).

The approximation of the frequency characteristics is performed in the same range of variables $\omega_0 = 0.01 \text{ s}^{-1}$, $\omega_N = 7 \text{ s}^{-1}$ as for LPF5. The number of approximation nodes is increased to N=50 to improve accuracy. Of all the N²=2500 abscesses (6), we determine the true ones by (7). By (9) we get $\tilde{N}_2^{low} = -1/0 \rightarrow -\infty$. The upper absolutely stable value of N₂ is obtained by (10): $\tilde{N}_2^{up} = 1/0.045 = 22.2$. The area of absolute stability represents a segment $0 \le N_2 \le 22.2$.

From a comparison of the regular $W_I(\omega)$ and modified $W_I^*(\omega)$ hodographs in Fig. 3 it can be seen that the stability regions of the formation differ in the linear and nonlinear modes of agents' functioning. Thus, the stability "in small" is determined by the abscissa of the point at which $W_I(\omega)=0$: $b_{up} = -0.034$, $N_2^{up} = 1/0.034 = 29.4$. The stability region in "small" is a segment $0 \le N_2 \le 29.4$. The general method for analyzing the stability of the form in the linear mode (for small perturbations) is based on the application of the Nyquist frequency criterion and the piecewise linear approximation of the hodograph.



Fig. 3. Regular and modified hodographs of the formation described by a complex oscillation-integrating LU of the 4th order

The upper limit value of N_2 in the nonlinear mode is significantly less than in the linear mode (1.32 times). This imposes restrictions on the choice of acceptable dynamic parameters of formation agents for large impacting perturbations.



 $\label{eq:result} \begin{array}{l} \textbf{REVISTA DE LA UNIVERSIDAD DEL ZULIA. } 3^a \mbox{ época. Año 11 N° 30, 2020} \\ \textbf{G. S. Vasilyev et al. /// Method for analyzing the stability of information transfer ...125-136} \end{array}$



Fig. 4. Dependences of the boundary stable coefficients N₂ on γ in the linear and non-linear mode for different filter types and orders

Fig. 4 also shows graphs of stable coefficients N_2 of the formations with LU described by bandpass (BPF) and notch (NF) filters of different orders (4, 6, 8, 10). Filter transfer functions have the form:

$$M_{2}^{LPF}(p) = 1/(1+Tp)^{I}, M_{2}^{HPF}(p) = (Tp)^{I}/(1+Tp)^{I},$$

$$M_{2}^{BPF}(p) = H_{LPF}(p)H_{HPF}(p) = (\gamma Tp)^{0.5I}/[(1+Tp)(1+\gamma Tp)]^{0.5I},$$

$$M_{2}^{NF}(p) = 1 - M_{2}^{BPF}(p).$$

Here, *T* is the time constant of the link in the LPF and HPF, γ is the ratio of the time constants of the LPF and HPF links in the BPF and NF. When the agents switch to the nonlinear mode of operation, the stability region of the formation significantly narrows from the top (by a factor of 2 for the 6th-order PF at γ =1 (Fig. 4d): $N_2^{up} = 64$ in linear mode, $\tilde{N}_2^{up} = 32$ in nonlinear mode). The results of the calculation of the lower limit of absolute stability for BPF coincide with the results of the calculation for the linear mode. Based on

absolutely stable values N_2 of a formation with LU at form of low-pass filter (LPF) and highpass filter (HPF) coincide with the values for stability "in the small" for any filter order to 10th inclusive.

Expressions are obtained that define the range of the loop amplification coefficient of the feedback circuit corresponding to the stable operation of the formation "as a whole" (for large amounts of influences). The use of CPLF allows us to study the absolute stability of formations of essentially nonlinear agents with different types and orders of the transfer function of the LU.

Conclusions

The relevance of the study of the absolute stability of formations of high-order nonlinear agents for the analysis of information exchange in UAV networks with an arbitrary amount of impacting perturbations (stability "as a whole") is noted. A method for analyzing the absolute stability of formations of nonlinear agents with different types and orders of the transfer function of LU is developed. The new approach is based on the use of the Popov frequency criterion and a piecewise linear approximation of the hodo-graph. The choice of a specific transfer function of the LU is carried out by simply substituting it's coefficients in the resulting expressions (1)-(10). A computational experiment was conducted to analyze the stability of a formation with LU of various types and orders from the 1st to the 10th. The conducted re-search revealed a significant difference in the calculated boundary stability coefficients of the formation in the linear and nonlinear mode, which confirms the need to analyze the nonlinear stability under the influence of strong destabilizing influences on the formation.

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