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Two-Dimensional Affine Transformation (AT2D) for Geodetic Velocity Interpolation utilizing SIRGAS-CON Weekly Solutions

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Abstract

Geodetic velocity models are critical for transforming coordinates from a reference epoch to an observation epoch. However, developing these models is often time-consuming, requiring frequent updates to maintain validity. To address the issue of its validity over time, this study examines the use of the Two-Dimensional Affine Transformation (AT2D) method for interpolating velocities at GNSS stations in South America. The AT2D relies on triangulation from surrounding SIRGAS-CON stations, which have well-known velocities, over a specific time interval. The resulting velocities from the AT2D method and VEMOS models were compared against their well-known reference velocities derived from SIRGAS-CON solutions, allowing the calculation of velocity residuals for quality assessment at fifty stations. In 82% of the residual comparisons, AT2D exhibited lower RMSE (±1.4 mm/year) than VEMOS models (±3.6 mm/year). The results show that the AT2D method effectively reproduces SIRGAS-CON velocities when GNSS stations are situated within the same tectonic plate. The primary advantage of the AT2D method is its rapid access and temporal flexibility, allowing for adjustments when abrupt jumps or data discontinuities occur. Therefore, the AT2D method proves efficient and accurate for calculating updated velocities in local or regional contexts relevant to surveying and geodetic engineering.
Keywords: Affine Transformation; Geodetic Velocities; GNSS; Interpolation; SIRGAS-CON.

Transformación Afín Bidimensional (AT2D) para Interpolación de Velocidades Geodésicas utilizando Soluciones Semanales SIRGAS-CON

Resumen

Los modelos de velocidades geodésicas son cruciales para la transformación de coordenadas de una época de referencia a una época de observación. Su desarrollo suele requerir mucho tiempo y actualizaciones frecuentes para mantener su validez. Para abordar el problema de su validez, este estudio examina el uso de la Transformación Afín Bidimensional (AT2D) para interpolar velocidades en estaciones GNSS en Sudamérica. La AT2D se basa en una

triangulación desde estaciones SIRGAS-CON circundantes, con velocidades conocidas, durante un intervalo de tiempo específico. Las velocidades resultantes del método AT2D y de los modelos VEMOS, sobre cincuenta estaciones, se compararon contra sus velocidades conocidas, lo que permitió el cálculo de residuos para la evaluación de la calidad. En el 82% de las comparaciones, la AT2D exhibió un RMSE menor (±1,4 mm/año) que los modelos VEMOS (±3,6 mm/año). Los resultados muestran que el método AT2D reproduce eficazmente las velocidades SIRGAS-CON cuando las estaciones se ubican en la misma placa tectónica. La AT2D ofrece una flexibilidad temporal que permite realizar ajustes ante saltos abruptos o discontinuidades en los datos GNSS. Además, la AT2D resulta eficiente y precisa para calcular velocidades actualizadas en contextos locales o regionales relevantes para la topografía y la ingeniería geodésica.
Palabras clave: Interpolación; GNSS; SIRGAS-CON; Transformación Afín; Velocidades Geodésicas.

Transformação Afin Bidimensional (AT2D) para Interpolação de Velocidades Geodésicas utilizando Soluções Semanais SIRGAS-CON

Resumo

Os modelos de velocidade geodésica são cruciais para a transformação de coordenadas de uma época de referência para uma época de observação. Seu desenvolvimento costuma ser demorado e requer atualizações frequentes para manter sua validade. Para abordar a questão da validade, este estudo examina o uso da Transformação Afim Bidimensional (AT2D) para interpolar velocidades em estações GNSS na América do Sul. A AT2D baseia-se na triangulação a partir de estações SIRGAS-CON vizinhas com velocidades conhecidas em um intervalo de tempo específico. As velocidades resultantes do método AT2D e dos modelos VEMOS, em cinquenta estações, foram comparadas com suas velocidades conhecidas, permitindo cálculos residuais para avaliação da qualidade. Em 82% das comparações, a AT2D apresentou um RMSE menor (±1,4 mm/ano) do que os modelos VEMOS (±3,6 mm/ano). Os resultados mostram que o método AT2D reproduz efetivamente as velocidades SIRGAS-CON quando as estações estão localizadas na mesma placa tectônica. O AT2D oferece flexibilidade temporal que permite ajustes em resposta a saltos abruptos ou descontinuidades nos dados GNSS. Além disso, o AT2D é eficiente e preciso para calcular velocidades atualizadas em contextos locais ou regionais relevantes para topografia e engenharia geodésica.
Palavras-chave: Interpolação; GNSS; SIRGAS-CON; Transformação Afim; Velocidades Geodésicas.

Introduction

Geodetic velocities refer to the rate of change of a point's position on the Earth's surface over time, expressed in a terrestrial reference frame and derived from continuous or episodic geodetic observations. Researchers typically derive geodetic velocities through various geodetic techniques, including Very Long Baseline Interferometry (VLBI), Global Navigation Satellite Systems (GNSS), Interferometric Synthetic Aperture Radar (InSAR), Light Detection and Ranging (LiDAR), Photogrammetry, leveling, tiltmeters, and total station measurements (Yan et al., 2022).

Geodetic velocities find broad applications in various geophysical and engineering contexts, including monitoring of plate tectonics, deformation processes, strain accumulation (and the prediction of potential seismic activity), structural stability analysis, sea-level studies (accounting for land movement in tide gauge analysis), glacier and ice sheet motion tracking, and regional/global deformation modeling. Furthermore, they are essential for updating coordinates in dynamic reference frames such as the International Terrestrial Reference Frame (Altamimi et al., 2023).

Within the framework of the Geodetic Reference System for the Americas (SIRGAS) (www.sirgas.ipgh.org) (Hoyer et al., 1998), the weekly coordinates from SIRGAS continuously operating network (SIRGAS-CON) provide a robust dataset with precisely known positions (referred to a specific reference epoch) and their changes over time (station velocities) (Sánchez, 2023; SIRGAS Analysis Centre at DGFI-TUM, 2024a). Based on SIRGAS reference frame solutions, a regularly updated velocity model for SIRGAS called VEMOS (Drewes et al., 2024) has been developed since 2003 (Drewes and Heidbach, 2005). However, generating velocity models takes considerable time, and ongoing geodynamic processes inherently limit their validity.

In contrast to continental models, such as VEMOS, this study seeks to develop an alternative method to calculate updated geodetic velocities, explicitly designed for a local or regional scale applications, including surveying and geodetic engineering projects. The proposed method utilizes SIRGAS-CON station coordinates to calculate velocities over a defined period, based on the principle that velocities are determined as the rate of coordinate change between two epochs within a consistent ITRF. This linear, uniform displacement model implicitly assumes the absence of significant trajectory anomalies such as abrupt jumps, antenna changes, receiver changes, or data discontinuities (Drewes, 2020).

To evaluate velocity interpolation, this study examines the Two-Dimensional Affine Transformation (AT2D) method (Ghilani, 2018), which is analogous to the Local TIN Interpolation (LTI) method employed in Digital Terrain Models (DTM) (Suárez et al., 2024). AT2D, widely used in geodesy, cartography, and geospatial analysis, enables point transformations on a two-dimensional plane, demonstrating effectiveness in both elevation and horizontal velocity field interpolation (Li et al., 2005).

In this research, data from 174 SIRGAS-CON stations distributed throughout South America were analyzed to estimate geodetic velocities in three temporal intervals between 2014.011 and 2024.011. Subsequently using the AT2D method to interpolate velocities at 50 target points (Pi) within triangular areas defined by three surrounding SIRGAS-CON stations (Sr) (Table 1) (Figure 1).

The results provide insights into the accuracy, constraints, and applicability of the AT2D method as an alternative for geodetic velocity estimation in a local context (range < 1,000 km), with a dense GNSS network, such as SIRGAS-CON, particularly in situations where a velocity model is not accessible.

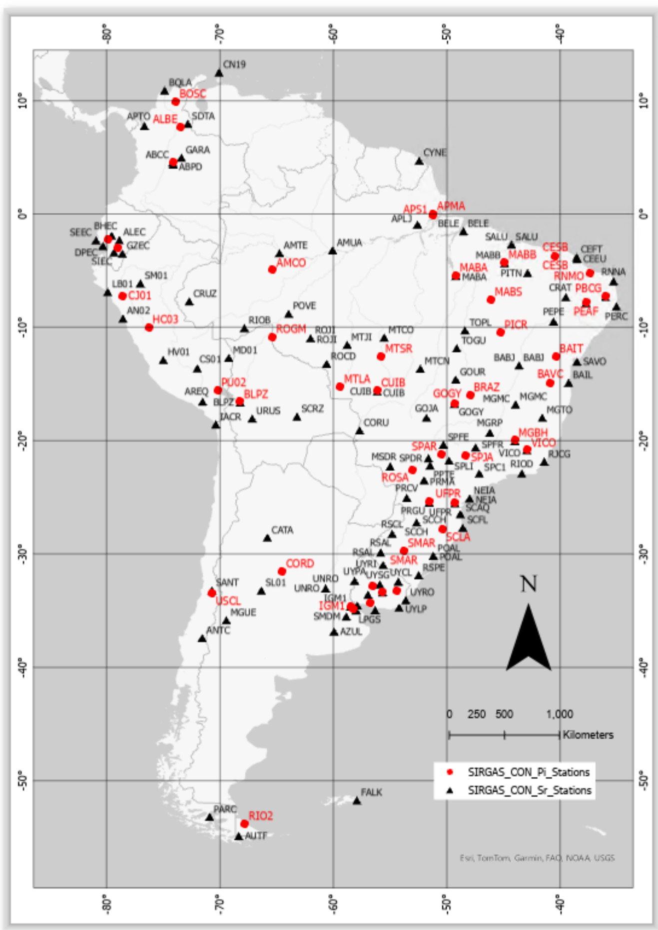

Figure 1. Map with selected SIRGAS-CON stations.

Table 1. Triangulations formed by selected SIRGAS-CON stations (P_i and S_r).

Period	P_i	S_r	P_i	S_r	P_i	S_r	P_i	S_r	P_i	S_r
2014.011-2017.011	AMCO	←AMTE	BLPZ	←MD01	GOGY	←MTCN	IGM1	←UNRO	MABB	←MABA
		←POVE		←URUS		←MGMC		←UYMO		←CRAT
		←CRUZ		←AREQ		←GOJA		←AZUL		←TOGU
	MABS	←MABA	MTSR	←MTCO	RIO2	←FALK	RNMO	←CEEU	ROGM	←RIOB
		←CRAT		←CUIB		←AUTF		←RNNA		←ROJI
		←TOGU		←ROCD		←PARC		←CRAT		←MD01
	ROSA	←PPTE	SCLA	←SCCH	UFPR	←NEIA	UYDU	←UYRI	VICO	←MGBH
		←PRCV		←SCFL		←SCFL		←UYRO		←RJCG
		←MSDR		←POAL		←PRGU		←UYMO		←RIOD
2017.011-2022.011	BAIT	←PEPE	BRAZ	←GOUR	CESB	←SALU	CORD	←CATA	CUIB	←MTJI
		←SAVO		←MGMC		←CEFT		←UNRO		←GOUR
		←BABJ		←GOGY		←PEPE		←SL01		←CORU
	MABA	←SALU	MGBH	←MGMC	PEAF	←CEEU	PICR	←TOPL	PRGU	←PRMA
		←TOPL		←VICO		←PBCG		←PEPE		←UFPR
		←AMUA		←MGRP		←PEPE		←BABJ		←SCCH
	SMAR	←SCCH	SPJA	←SPFR	UYPT	←UYPA	UYSJ	←UYFS	UYTT	←UYCL
		←RSPE		←SPC1		←UYSG		←UYMO		←UYLP
		←RSAL		←SPLI		←UYFS		←UYCO		←UYDU
2022.011-2024.011	AGGO	←IGM1	ALBE	←SDTA	APMA	←CYNE	APS1	←CYNE	BAVC	←BABJ
		←LPGS		←GARA		←BELE		←BELE		←BAIL
		←SMDM		←APTO		←APLJ		←APLJ		←MGTO
	BOGT	←GARA	BOSC	←CN19	CESB	←SALU	CJ01	←SM01	CUEC	←ALEC
		←ABPD		←SDTA		←CEFT		←AN02		←GZEC
		←ABCC		←BQLA		←MABB		←LB01		←SIEC
	GQEC	←BHEC	HC03	←SM01	MTLA	←ROJI	PBCG	←RNNA	PU02	←CS01
		←DPEC		←HV01		←CUIB		←PERC		←BLPZ
		←SEEC		←AN02		←SCRZ		←PEAF		←IACR
	SMAR	←RSCL	SPAR	←SPFE	UFPR	←NEIA	USCL	←SANT	UYSJ	←UYFS
		←POAL		←SPLI		←SCAQ		←MGUE		←UYMO
		←RSAL		←SPDR		←PRGU		←ANTC		←UYCO

Data and methodology

This study utilized three distinct data sources and employed three specialized programs for data processing:

The authors extracted the reference frame, epoch, latitude, longitude, and ellipsoidal height data from SIRGAS weekly ellipsoidal position files (*.PLH), for the period 2014.011 to 2024.011 (Sánchez et al., 2022).

SIRGAS weekly position differences (*.NEU) files, which include the reference frame, epoch in decimal years and position differences (ΔE, ΔN), were also extracted, for the period 2014.011 to 2024.011 (Sánchez et al., 2022). All SIRGAS weekly products presented and discussed in this paper are publicly available at the following website (SIRGAS Analysis Centre at DGFI-TUM, 2024b): https://www.sirgas.org/en/stations/station-list/.

*.TXT files of the VEMOS2017 (Drewes and Sánchez, 2020) and VEMOS2022 (Drewes et al., 2024) velocity models. VEMOS2017 was derived from pointwise station velocities inferred at 515 geodetic sites from January 1, 2014, to January 28, 2017, using a geodetic least-squares collocation approach with empirically determined covariance functions. The average uncertainty of VEMOS2017 is ±1.0 mm/yr in the north-south direction and ±1.7 mm/yr in the east-west direction. VEMOS2022 was computed using the same methods as the previous VEMOS2017, covers the period from February 1, 2017, to April 30, 2022, and its average uncertainty is assessed to be ±0.8 mm/yr in the north-south direction and ±1.3 mm/yr in the east-west direction (Sánchez et al., 2022). All VEMOS models presented and discussed in this paper are publicly available at the following website (SIRGAS Analysis Centre at DGFI-TUM, 2024c): https://www.sirgas.org/en/velocity-model.

Velocities interpolation programs SGC_VEMOS2017_Interp.exe and SGC_VEMOS2022_Interp.exe (Suárez, 2023) based on bilinear models from Python's NumPy library (Harris et al., 2020). Velocities interpolation program using the Two-Dimensional Affine Transformation SGC_SIRGAS-AT2D_Interp.exe (Suárez, 2023) based on Python's NumPy library (Harris et al., 2020).

This research examines the Two-Dimensional Affine Transformation (AT2D) method (Ghilani, 2018), to interpolate the horizontal velocity components of any point to be interpolated (Pi) within the area of a triangle formed by three surrounding SIRGAS-CON reference stations (Sr) (Figure 2). The geodetic coordinates (Xr, Yr), and velocities (VEr, VNr) are known for a defined time interval (tf - ti), within the same ITRF. Figure 2 presents an example of triangulation involving three surrounding SIRGAS-CON reference stations—Sr, CUIB, ROJI, and SCRZ—with the interpolation target station (Pi), MTLA, positioned at the center. The distances between these stations range from 350 to 560 km.

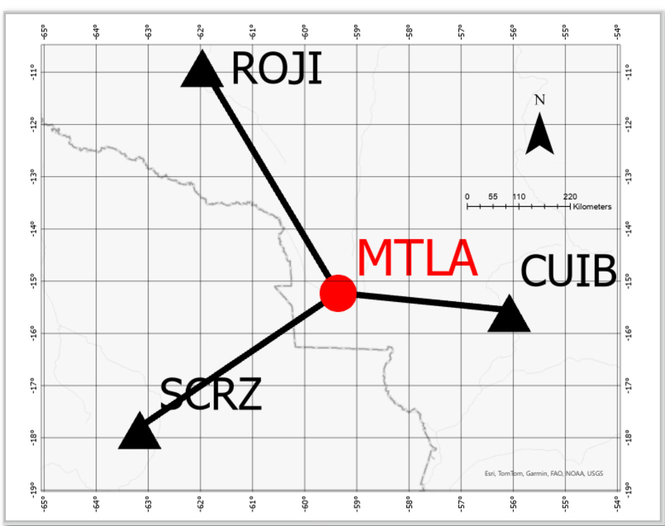

Figure 2. Example of triangulation: (Sr: CUIB, ROJI, SCRZ) and (Pi: MTLA).

In this study, the authors selected SIRGAS-CON stations based on three main criteria: geodynamics, geometrics, and data availability. The authors located all stations (Pi and Sr) on the same tectonic plate to ensure consistent velocity interpolation. Furthermore, for maximum accuracy, the configuration of the selected SIRGAS-CON reference stations (Sr) was as close as possible to the Pi station, with good symmetry and proportionality. The selected stations should form triangles having a ratio between the perimeter and the square root of the enclosed area [P/√A ≈ 4.5590] (Feder, 1988). This criterion improves the geometric stability of the transformation and reduces potential interpolation errors. Finally, the chosen reference stations (Sr) must not exhibit data jumps or discontinuities during the time interval (tf - ti) of interest.

With the reference stations selected, the authors proceed with their velocity's calculation. It is important to highlight that the authors did not derive velocities through a regression model of the coordinate time series. Instead, the authors calculated the SIRGAS-CON stations' velocities over a defined period corresponding to the rate of change of their coordinates (position differences) between two observation epochs within the same ITRF (Drewes, 2020). The calculation of horizontal velocities (VEr, VNr) of the SIRGAS-CON reference stations (Sr) was based on weekly position difference files (*.NEU), using the coordinate changes (ΔEr, ΔNr) between two observation epochs (tf and ti), representing discrete positional shifts over a defined time interval. The governing equations are shown below (Eq. 1, 2).

$$VE_r = \frac{(\Delta E_f - \Delta E_i)}{(t_f - t_i)} \tag{1}$$

$$VN_r = \frac{(\Delta N_f - \Delta N_i)}{(t_f - t_i)} \tag{2}$$

The authors interpolated the velocities at the point of interest (P_i) by using the Two-Dimensional Affine Transformation (AT2D), which applies two translations of the origin, and a rotation about the origin, plus a small nonorthogonality correction between the X and Y axes (Ghilani, 2018). AT2D utilizes six parameters (a, b, c, d, e, f), including two translations (c, f), two independent scales, one rotation, and one distortion, allowing for modeling not only rotations and translations but also stretching and distortions in two different directions. X and Y are the well-known coordinates (longitude and latitude) extracted from *.PLH files. Equations (3) and (4) show the observation equations of the AT2D model.

$$aX + bY + c = VE \tag{3}$$

$$dX + eY + f = VN \tag{4}$$

When the number of equations equals the number of unknowns, and the equations are linearly independent, meaning there is no redundancy among them, and the coefficient matrix is invertible, it ensures the existence of a unique solution (Lay *et al.*, 2016). In this case, a determined solution of the linear equations is possible with a minimum of three reference stations (S_r). The matrix notation of the observation equation system AX = L is shown below (Eq: 5):

$$\begin{bmatrix} X_1 & Y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_3 & Y_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} VE_1 \\ VN_1 \\ VE_2 \\ VN_2 \\ VE_3 \\ VN_3 \end{bmatrix} \tag{5}$$

Once the authors explicitly solve the observation equation system AX = L by calculating $X=A^{-1}L$ using matrix inversion, and all its unknown parameters (a, b, c, d, e, f) are determined. The authors interpolate the velocities at the point of interest (P_i) with known coordinates (X_i, Y_i) using equations (6) and (7).

$$VE_i = aX_i + bY_i + c \tag{6}$$

$$VN_i = dX_i + eY_i + f \tag{7}$$

For this study, the authors selected 50 triangles (Table 1), each with their respective interior point to be interpolated (P_i) and its three surrounding stations (S_r), all belonging to the SIRGAS-CON network, for a total of 200 stations (174 unique stations) (Figure 1). The authors applied the AT2D method to interpolate the velocities at the 50 stations (P_i), which are also well-known from SIRGAS-CON weekly solution, allowing the calculation of their velocity residuals (Eq. 8, 9) and their respective Root Mean Square Error (RMSE) for each velocity component (Eq. 10, 11).

$$\delta VE_i = VE_{i(AT2D)} - VE_{i(SIRGAS-CON)} \tag{8}$$

$$\delta VN_i = VN_{i(AT2D)} - VN_{i(SIRGAS-CON)} \tag{9}$$

$$RMSE_{VE} = \sqrt{\frac{\sum_{i=1}^{n}(\delta VE_i)^2}{n}} \tag{10}$$

$$RMSE_{VN} = \sqrt{\frac{\sum_{i=1}^{n}(\delta VN_i)^2}{n}} \tag{11}$$

Results and Discussion

A comparative analysis was conducted on three interpolation techniques —AT2D, VEMOS2017, and VEMOS2022— to evaluate their effectiveness in estimating velocity components. The AT2D method was the sole technique developed within the scope of this study. VEMOS models were incorporated as external data products for comparative purposes. The authors benchmarked the residuals from each method against the established SIRGAS-CON velocity values. The authors performed the first comparative analysis of velocities for the period 2014.011–2017.011, simultaneous with VEMOS2017, using fifteen triangular configurations formed by SIRGAS-CON stations [15(P_i) + 45(S_r)]. In this analysis, the distance between the P_i and its corresponding S_r varies from 120 to 870 km (Figure 3).

From Table 2, the AT2D method exhibited the lowest RMSE (VN: ±1.1 mm/yr, VE: ±1.4 mm/yr) among three velocity interpolation methods (AT2D, VEMOS2017, VEMOS2022) compared against reference SIRGAS-CON velocities. In addition, AT2D's direction (sign) of the interpolated velocities matches the SIRGAS-CON velocities for all fifteen stations. VEMOS17 failed at station BLPZ, and VEMOS2022 failed at stations BLPZ, RIO2, and RNMO.

An additional external review of the SIRGAS2022 positions and velocities files (*.CRD) reported the following velocities: [BLPZ, VN: 13.67, VE: 1.66 mm/year]; [RIO2, VN: 11.71, VE: 4.42 mm/year]; [RNMO, VN: 15.8, VE: -4.08 mm/year]. These velocities are more consistent with those in AT2D, except for BLPZ. All three stations share common discontinuity issues: BLPZ was affected by the Iquique Quake on 4/1/2014; RIO2 experienced an antenna switch on 2/20/2020, and a receiver upgrade on 6/30/2020; and RNMO experienced an antenna switch on 3/28/2017.

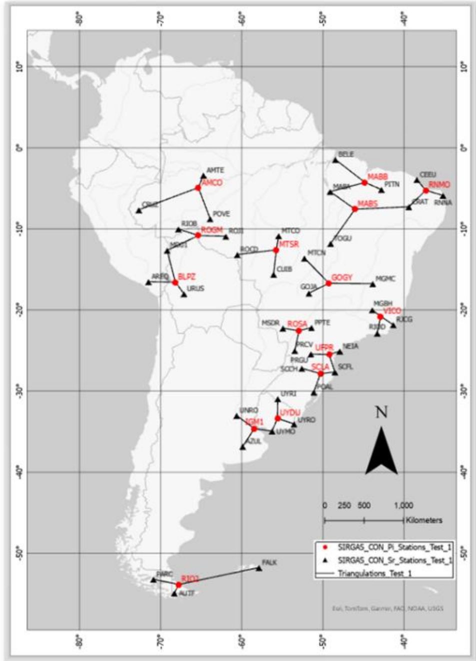

Figure 3. SIRGAS-CON stations [15(P_i) + 45(S_r)], period 2014.011–2017.011.

Table 2. Interpolated velocities and residuals [mm/yr] (period 2014.011-2017.011).

Units:mm/yr	SIRGAS-CON		AT2D		Res.		VEMOS2017		Res.		VEMOS2022		Res.	
P_i	VN_i	VE_i	VN_i	VE_i	δVN_i	δVE_i	VN_i	VE_i	δVN_i	δVE_i	VN_i	VE_i	δVN_i	δVE_i
AMCO	11.31	-2.31	11.29	-3.18	-0.02	-0.87	11.86	-2.89	0.55	-0.58	13.30	-3.93	0.54	0.47
BLPZ	8.47	-2.2	8.23	-2.44	-0.24	-0.24	11.80	**2.94**	3.33	5.14	12.86	**2.33**	4.39	4.53
GOGY	11.88	-4.39	11.26	-4.83	-0.62	-0.44	12.59	-2.59	0.71	1.8	11.48	-2.62	-0.3	1.48
IGM1	11.78	-4.1	11.9	-3.43	0.12	0.67	13.10	-1.93	1.32	2.17	12.73	-3.68	0.85	0.71
MABB	12.76	-4.4	12.62	-4.4	-0.14	0	13.04	-2.87	0.28	1.53	12.15	-2.94	0.83	-0.08
MABS	12.07	-6.28	12.85	-3.27	0.78	3.01	12.55	-3.31	0.48	2.97	12.26	-3.39	0.95	-1.08
MTSR	12.71	-4.17	11.2	-2.91	-1.51	1.26	12.82	-2.75	0.11	1.42	11.56	-2.02	0.98	-0.18
RIO2	11.28	4.73	11.63	6.24	0.35	1.51	14.43	6.71	3.15	1.98	11.88	**-3.16**	0.3	1.22
RNMO	13.11	-5.72	11.47	-4.41	-1.64	1.31	14.10	-3.67	0.99	2.05	12.03	**5.38**	0.75	0.65
ROGM	10.58	-1.84	11.61	-2.17	1.03	-0.33	11.66	-1.27	1.08	0.57	13.40	-3.74	0.69	0.43
ROSA	11.58	-4.38	12.22	-3.41	0.64	0.97	12.45	-1.82	0.87	2.56	14.02	-4.55	0.91	1.17
SCLA	11.32	-2.86	11.9	-3.4	0.58	-0.54	13.62	-1.81	2.3	1.05	11.72	-2.68	2.07	-1.1
UFPR	13.21	-3.27	10.91	-3.38	-2.3	-0.11	13.37	-2.33	0.16	0.94	12.61	-3.36	-0.6	-0.09
UYDU	9.65	-1.58	11.45	-2.7	1.8	-1.12	13.07	-1.33	3.42	0.25	12.42	-4.77	1.91	-0.47
VICO	10.51	-4.3	10.68	-7.4	0.17	-3.1	12.65	-3.33	2.14	0.97	13.20	-4.30	1.13	1.98
				RMSE(mm/yr):	**±1.1**	**±1.4**	**RMSE(mm/yr):**		**±1.8**	**±2.1**	**RMSE(mm/yr):**		**±1.5**	**±1.5**

A second comparative analysis was conducted for the period 2017.011–2022.011, concurrent with VEMOS2022, using a different data set of fifteen pre-defined triangular configurations of SIRGAS-CON stations [15(P_i) + 45(S_r)]. The distance between the P_i and its corresponding S_r varies from 66 to 1,235 km (Figure 4).

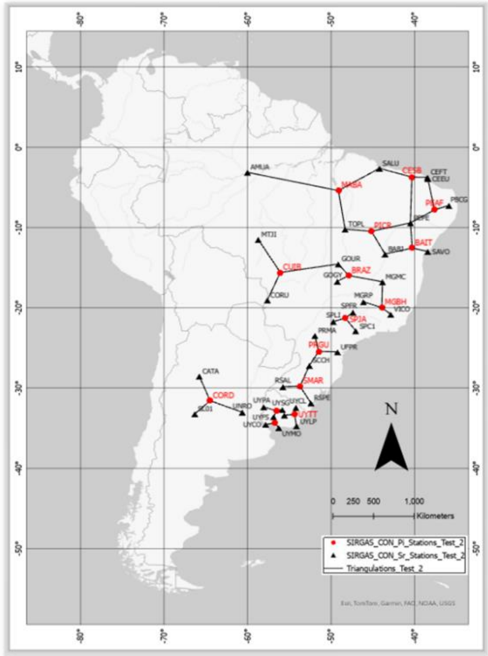

Figure 4. SIRGAS-CON stations [15(P_i) + 45(S_r)], period 2017.011-2022.011.

Table 3. Interpolated velocities and residuals [mm/yr] (period 2017.011-2022.011).

Units:mm/yr	SIRGAS-CON		AT2D		Res.		VEMOS2017		Res.		VEMOS2022		Res.	
P_i	VNi	VEi	VN_i	VE_i	δVN_i	δVE_i	VN_i	VE_i	δVN_i	δVE_i	VN_i	VE_i	δVN_i	δVE_i
BAIT	11.77	-4.66	12.71	-4.84	0.94	-0.18	12.89	-3.29	1.12	1.37	11.52	-2.83	0.42	-0.06
BRAZ	12.41	-4.79	12.42	-3.87	0.01	0.92	12.69	-2.65	0.28	2.14	11.98	-2.61	0.04	0.63
CESB	12.21	-5.55	10.61	-5.12	-1.6	0.43	14.02	-3.84	1.81	1.71	10.10	-3.03	0.08	-0.96
CORD	10.02	-2.07	9.85	-1.87	-0.17	0.2	11.71	-2.74	1.69	-0.67	12.59	-3.61	0.42	0.17
CUIB	10.58	-3.27	11.99	-4.11	1.41	-0.84	12.27	-2.5	1.69	0.77	11.96	-2.74	0.5	0.2
MABA	12.19	-6.08	11.11	-5.17	-1.08	0.91	13.12	-3.23	0.93	2.85	13.10	-4.06	0.49	1.26
MGBH	11.29	-3.01	12.73	-3.67	1.44	-0.66	12.87	-3.42	1.58	-0.41	11.59	-2.63	0.04	0.21
PEAF	12.87	-5.48	12.28	-5.22	-0.59	0.26	13.1	-3.8	0.23	1.68	13.04	-5.28	0.17	0.2
PICR	12.61	-5.32	12.25	-4.92	-0.36	0.4	12.94	-2.26	0.33	3.06	12.79	-3.80	0.38	0.99
PRGU	12.03	-3.21	12.19	-4.6	0.16	-1.39	14.09	-1.79	2.06	1.42	12.71	-4.16	0.94	0.5
SMAR	11.94	-3.24	11.93	-3.18	-0.01	0.06	13.55	-1.9	1.61	1.34	12.74	-2.83	0.71	0.38
SPJA	12.17	-3.78	12.44	-3.73	0.27	0.05	12.67	-2.44	0.5	1.34	13.01	-3.80	0.82	2.28
UYPT	11.1	-2.77	11.09	-2.75	-0.01	0.02	12.9	-1.42	1.8	1.35	12.69	-4.42	1.4	-1.41
UYSJ	11.55	-2.84	10.97	-2.74	-0.58	0.1	13.11	-1.62	1.56	1.22	12.70	-3.43	2.12	-0.16
UYTT	11.46	-2.94	11.43	-2.64	-0.03	0.3	13.15	-1.64	1.69	1.3	13.06	-4.56	0.85	0.99
				RMSE(mm/yr):	**±0.8**	**±0.6**	**RMSE(mm/yr):**		**±1.4**	**±1.7**	**RMSE(mm/yr):**		**±0.8**	**±0.9**

From Table 3, the AT2D method showed the lowest RMSE in both velocity components: VN: ±0.8 mm/yr and VE: ±0.6 mm/yr, while the highest RMSE was reported by the VEMOS2017 model. The interpolated velocities from VEMOS2022 reported second-best results, which is expected because this model was originally computed using data from the same period. In this case, the direction (sign) of the interpolated velocities for the three methods matches the SIRGAS-CON velocities for all fifteen stations.

The third and last comparative analysis was done for the period 2022.011-2024.011 (non-concurrent with VEMOS), using another data set of twenty pre-defined triangular configurations of SIRGAS-CON stations [$20(P_i)$ + $60(S_r)$]. The distance between the P_i and its corresponding S_r varies from 6 to 560 km (Figure 5).

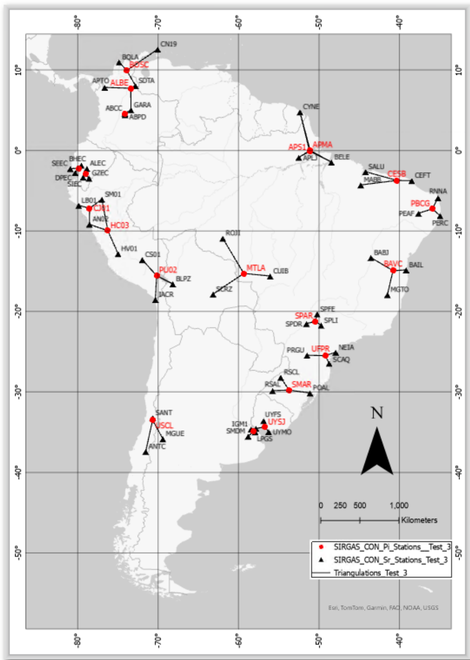

Figure 5. SIRGAS-CON stations [$20(P_i)$ + $60(S_r)$], period 2022.011-2024.011.

From Table 4, the AT2D method reported the lowest RMSE in both velocity components: VN: ±0.9 mm/yr and VE: ±0.7 mm/yr, while the highest RMSE was reported by the VEMOS2017 model. AT2D's and VEMOS2022's direction (sign) of the interpolated velocities aligns with the SIRGAS-CON velocities for all twenty stations. VEMOS17 failed at the CJ01 and CUEC stations. Both stations have similar discontinuity issues: the Navarro Earthquake on 5/26/2019 impacted CJ01, and CUEC underwent four antenna switches between 2014 and 2020 and was affected by three earthquakes, namely Quake Muisne/Pedernales on 4/16/2016, Quake Palora on 2/22/2019, and Quake Navarro on 5/26/2019.

Table 4. Interpolated velocities and residuals [mm/yr] (period 2022.011-2024.011).

Units:mm/yr	SIRGAS-CON		AT2D		Res.		VEMOS2017		Res.		VEMOS2022		Res.	
P_i	VN_i	VE_i	VN_i	VE_i	δVN_i	δVE_i	VN_i	VE_i	δVN_i	δVE_i	VN_i	VE_i	δVN_i	δVE_i
AGGO	9.03	-3.31	10.65	-2.54	1.62	0.77	13.15	-1.9	4.12	1.41	11.55	-2.43	2.52	0.88
ALBE	12.94	4.82	11.93	4.72	-1.01	-0.1	13.58	6.62	0.64	1.8	15.26	4.66	2.32	-0.16
APMA	11.33	-5.42	11.4	-5.21	0.07	0.21	12.23	-3	0.9	2.42	12.92	-4.51	1.59	0.91
APS1	11.66	-5.69	11.4	-5.19	-0.26	0.5	12.23	-2.99	0.57	2.7	12.91	-4.51	1.25	1.18
BAVC	10.42	-5.47	11.51	-5.91	1.09	-0.44	13	-3.11	2.58	2.36	12.76	-4.06	2.34	1.41
BOGT	11.57	0.81	11.56	0.49	-0.01	-0.32	13.57	2.45	2	1.64	14.75	0.00	3.18	-0.81
BOSC	14.02	6.15	12.78	7.09	-1.24	0.94	12.83	10.42	-1.19	4.27	15.13	8.49	1.11	2.34
CESB	11.51	-6.06	10.5	-5.9	-1.01	0.16	14.02	-3.84	2.51	2.22	13.06	-4.56	1.55	1.5
CJ01	5.17	-1	5.44	-1.97	0.27	-0.97	8.37	**3.75**	3.2	4.75	9.07	-0.96	3.9	0.04
CUEC	6.96	-0.5	6.86	-2.21	-0.1	-1.71	9.52	**0.36**	2.56	0.86	8.79	-0.66	1.83	-0.16
GQEC	6.78	2.47	7.58	1.51	0.8	-0.96	9.92	1.36	3.14	-1.11	9.91	2.77	3.13	0.3
HC03	9.99	2	7.5	2.27	-2.49	0.27	12.15	8.38	2.16	6.38	12.53	5.48	2.54	3.48
MTLA	10.43	-4.4	9.8	-4.18	-0.63	0.22	11.53	-2.13	1.1	2.27	12.14	-3.12	1.71	1.28
PBCG	11.64	-6.48	10.81	-5.89	-0.83	0.59	13.45	-4.11	1.81	2.37	13.18	-5.14	1.54	1.34
PU02	9.69	2.42	9.24	1.83	-0.45	-0.59	12.49	7.47	2.8	5.05	12.58	6.19	2.89	3.77
SMAR	11.89	-4.49	11.64	-4.61	-0.25	-0.12	13.55	-1.9	1.66	2.59	11.98	-2.61	0.09	1.88
SPAR	11.85	-4.93	11.27	-5.86	-0.58	-0.93	12.66	-2.18	0.81	2.75	12.57	-3.31	0.72	1.62
UFPR	11.71	-4.77	11.25	-4.87	-0.46	-0.1	13.37	-2.33	1.66	2.44	12.61	-3.36	0.9	1.41
USCL	12.75	12.35	13.48	13.86	0.73	1.51	18.48	3.06	5.73	-9.29	16.70	13.86	3.95	1.51
UYSJ	11.32	-3.13	11.33	-3.27	0.01	-0.14	13.11	-1.62	1.79	1.51	11.59	-2.63	0.27	0.5
				RMSE(mm/yr):	**±0.9**	**±0.7**	**RMSE(mm/yr):**		**±2.5**	**±3.6**	**RMSE(mm/yr):**		**±2.2**	**±1.6**

Across all tests, the AT2D method consistently exhibits the lowest RMSE values compared to VEMOS2017 and VEMOS2022. Out of fifty comparisons (Tables 2–4), forty-one stations (82%) show lower RMSE values with AT2D. The maximum RMSE values observed were ±1.4 mm/year for AT2D, ±3.6 mm/year for VEMOS2017, and ±2.2 mm/year for VEMOS2022. Conversely, the minimum RMSE values were ±0.6 mm/year, ±1.4 mm/year, and ±0.8 mm/year, respectively. The AT2D method achieves accuracy better than ±1 mm/year with SIRGAS-CON data collected after 2017 due to fewer discontinuities compared to earlier datasets.

The precision of VEMOS models decreases notably when interpolations occur outside the time intervals for which the models were originally developed, limiting their temporal applicability. In contrast, the AT2D method offers considerable temporal flexibility, allowing users to adjust, expand, or shift time intervals, which is particularly advantageous when handling discontinuities, especially in seismic active regions, since the AT2D method is based on SIRGAS weekly coordinate solutions (Sánchez and Drewes, 2020).

Spatial separation between interpolated stations (P_i) and reference stations (S_r) does not significantly influence AT2D performance. However, suboptimal geometric arrangements (triangles with perimeter-to-area ratios [$P/\sqrt{A}$] below 4.5590) or data discontinuities can significantly reduce AT2D efficacy. Furthermore, the AT2D method should not be used for interpolation beyond triangular regions, such as coastal zones, nor in areas close to active tectonic plate boundaries or regions experiencing recent seismic activity.

The interpolation of geodetic velocities using the AT2D method is dependent on all participating stations (P_i and S_r) being located on the same tectonic plate and same ITRF to ensure consistency in velocity estimates.

In scenarios involving geodynamic processes, such as co-seismic or post-seismic displacements, more robust methods like Trajectory Models (TM) (Bevis and Brown, 2014) or Least Squares Collocation (LSC) (Gómez et al., 2016) are recommended.

Conclusions

- The AT2D is an effective technique for calculating updated geodetic velocities, particularly in local or regional surveying and geodetic engineering applications, in a relatively rapid way. AT2D does not compete with VEMOS models on a large or continental scale.

- The AT2D method reliably and effectively replicates SIRGAS-CON velocities and displacements, especially when GNSS stations are situated within the stable interior of a tectonic plate. On the contrary, this method is unsuitable for stations located near active plate boundaries or within regions that have recently been affected by seismic activity.

- The AT2D approach is recommended for regions equipped with a robust and dense GNSS continuous observation network, especially when precise velocity models are not available. Nevertheless, it is not appropriate for areas with sparse GNSS networks, such as Chile, Paraguay, and Venezuela.

- The AT2D method exceeds conventional grid-based models in its ability to facilitate users in selecting the most appropriate nearby stations and defining a suitable, and notably, current time interval. The temporal flexibility inherent in the AT2D method permits the adjustment of time intervals, which proves advantageous for managing GNSS data characterized by sudden jumps or discontinuities.

- While this study utilized three surrounding reference stations (Sr) for velocity interpolation, the AT2D method inherently allows incorporation of additional stations. Further research is recommended to explore whether incorporating more stations enhances interpolation robustness.

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References

Altamimi, Z., Rebischung, P., Collilieux, X., Métivier, L., Chanard, K. (2023): *ITRF2020: an augmented reference frame refining the modeling of nonlinear station motions*. Journal of Geodesy. doi:10.1007/s00190-023-01738-w.

Bevis, M., Brown, A. (2014). *Trajectory models and reference frames for crustal motion geodesy*. Journal of Geodesy. Volume 88, pages 283–311. doi: 10.1007/s00190-013-0685-5.

Drewes, H., Heidbach, O. (2005). *Deformation of the South American crust estimated from finite element and collocation methods', in Sansò, F. (ed.)*. A Window on the Future of Geodesy. IAG Symposia, vol. 128, pp. 544–549. Springer, Berlin, Heidelberg. doi: 10.1007/3-540-27432-4_92.

Drewes, H. (2020). *Modelar el movimiento de la superficie terrestre. Velocidades continuas y coordenadas por etapas*. Technische Universität München (TUM), International Association of Geodesy (IAG), Sistema de Referencia Geodésico para las Americas (SIRGAS). Webinar SIRGAS, 28 August 2020. Available at: www.sirgas.ipgh.org/docs/Boletines/2020_Drewes_Webinar_VEMOS.pdf.

Drewes, H., Sánchez, L. (2020). *Velocity model for SIRGAS 2017: VEMOS2017*. PANGAEA. Technische Universität München, Deutsches Geodätisches Forschungsinstitut (DGFI-TUM), IGS RNAAC SIRGAS. [Online]. doi: 10.1594/PANGAEA.912350.

Drewes, H., Seitz, M., Sánchez, L. (2024). *Realisation of the Non-Rotating Terrestrial Reference Frame by an Actual Plate Kinematic and Crustal Deformation Model (APKIM2020).* International Association of Geodesy Symposia. Springer, Berlin, Heidelberg. doi:10.1007/1345_2024_276.

Feder, J. (1988). *The Perimeter-Area Relation*. Fractals: Physics of Solids and Liquids. Springer, Boston, MA. doi:10.1007/978-1-4899-2124-6_12.

Ghilani, C. (2018). *Adjustment Computations: Spatial Data Analysis.* 6th edn. John Wiley & Sons, Inc. Hoboken, New Jersey.

Gómez, D., Piñón, D., Smalley, R., Bevis, M., Cimbaro, S., Lenzano, L., Barón, J. (2016). Reference frame access under the effects of great earthquakes: a least squares collocation approach for non-secular post-seismic evolution. Journal of Geodesy, Volume 90, Issue 3, pp. 263-273. DOI:10.1007/s00190-015-0871-8.

Harris, C., Millman, K., van der Walt, S., Gommers, R., Virtanen, P., Cournapeau, D., Wieser, E., Taylor, J., Berg, S., Smith, N., Kern, R., Picus, M., Hoyer, S., Kerkwijk, M., Brett, M., Haldane, A., Fernández del Río, J., Wiebe, M., Peterson, P., Oliphant, T. (2020). *Array programming with NumPy*. Nature 585(7825), pp. 357–362. doi:10.1038/s41586-020-2649-2.

Hoyer, M., Arciniegas, S., Pereira, K., Fagard, H., Maturana, R., Torchetti, R., Drewes, H., Kumar, M., Seeber, G. (1998). *The Definition and Realization of the Reference System in the SIRGAS Project', in Brunner, F.K. (ed.)*. Advances in Positioning and Reference Frames. International Association of Geodesy Symposia, vol. 118. Springer, Berlin, Heidelberg.

Lay, D., Lay, S., McDonald, J. (2016). *Linear Algebra and Its Applications*. 5th edn. Pearson.

Li, Z., Zhu, C., Gold, C., Wu, H., Fritsch, D. (2005). *Digital Terrain Modeling: Principles and Methodology*. CRC Press.

Sánchez, L. (2023). *SIRGAS Regional Network Associate Analysis Centre, Technical Report 2022*. IGS Technical Report 2022. doi:10.48350/179297.

Sánchez, L. Drewes, H. (2020). *Geodetic monitoring of the variable surface deformation in Latin America*. International Association of Geodesy Symposia Series, vol. 152. Springer, Cham. doi: 10.1007/1345_2020_91

Sánchez, L., Drewes, H., Kehm, A., Seitz, M. (2022). *SIRGAS reference frame analysis at DGFI-TUM*. Journal of Geodetic Science, 12(1), pp. 92–119. doi:0.1515/jogs-2022-0138.

SIRGAS Analysis Centre at DGFI-TUM. (2024a). *SIRGAS continuously operating network*. Available at: www.sirgas.org/en/sirgas-realizations/sirgas-con-network/.

SIRGAS Analysis Centre at DGFI-TUM. (2024b). *SIRGAS continuously operating stations*. Available at: www.sirgas.org/en/stations/station-list/.

SIRGAS Analysis Centre at DGFI-TUM. (2024c). *VEMOS: Velocity model for SIRGAS*. Available at: www.sirgas.org/en/velocity-model/.

Suárez, H. (2023). *SIGGMA GEODETIC CALCULATOR – SGC*. Sociedad de Ingenieros Geodestas, Geomáticos y Agrimensores de Venezuela – SIGGMA. Available at: www.siggma.xyz/sgc/.

Suárez, H., Arenas, I., Cano, N., Marcano, P., Diaz, M., Martínez, A. (2024). *Local TIN Interpolation (LTI) for the calculation of horizontal velocity components, based on weekly time series from SIRGAS-CON stations.* Sistema de Referencia Geodésico para las Américas (SIRGAS). Simposio SIRGAS 2024, Bogotá, Colombia, 18-21 de noviembre de 2024.

Yan, Y., Li, M., Dai, L., Guo, J., Dai, H., Tang, W. (2022). *Construction of "Space-Sky-Ground" Integrated Collaborative Monitoring Framework for Surface Deformation in Mining Area*. Remote Sensing, 14, 840. doi: 10.3390/rs14040840.

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